

# Electrical Design and Musical Applications of an Unconditionally Stable Combination Voltage Controlled Filter/Resonator

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A new voltage-controlled filter designed for the ARP synthesizer is a combination low-pass, high-pass, and band-reject filter capable of obtaining  $Q$  as high as 500. The filter is unconditionally stable and the  $Q$  can be controlled from  $\frac{1}{2}$  to 500 by an external control voltage. The center frequency of the filter can be varied continuously over 20 octaves on a 1-volt per octave response to an external control signal.

**INTRODUCTION:** Since the acceptance of electronic music synthesizers in recording studios, bands, and educational institutions, the demand for a wider range of sound timbres has increased greatly. The filter to be described here was initially designed for use in the modular ARP<sup>1</sup> synthesizer, to extend the gamut of sound spectra available. The main design objectives were 1) to achieve a very high but stable resonance factor ( $Q$ ), 2) to provide high-pass, band-pass, and band-reject (notch) capability as well as the already available low-pass function, and 3) to allow accurate and stable voltage control of both the center frequency  $F_c$  and  $Q$  in an exponential manner.

These goals have been achieved in what is called the ARP module 1047 multimode filter/resonator. In addition, many new applications have been realized, such as accurate real-time spectral analysis, biological signal

processing, speech analysis and synthesis, and analog computation, to mention a few.

## BACKGROUND

Music synthesizers typically generate waveforms with rich but uniform harmonic structures, such as sawtooth or pulse waves. These are then fed into a filter, whose amplitude-frequency response modifies the amplitude relation of the harmonics, creating a change in the timbre ("tone") of the sound (Fig. 1).

Since the filter is the major timbre controlling element, it is obviously desirable to 1) have a wide variety of frequency response shapes available, and 2) be able to rapidly change the filter's parameters through voltage control; that is, to vary dynamically the timbre of the sound.

However, most synthesizers rely on a voltage-controlled low-pass filter with relatively sharp (24 dB per octave) cutoff and moderate resonance capability (20 dB peak, or a  $Q$  of 10). Although this response is certainly useful, most natural sounds do not resemble the harmonic structures attainable with this filter. Rather, they are the result of impressing a simple waveform from, for example,

\* Presented April 29, 1971, at the 40th Convention of the Audio Engineering Society, Los Angeles.

<sup>1</sup> ARP is a trademark of Tonus, Inc.

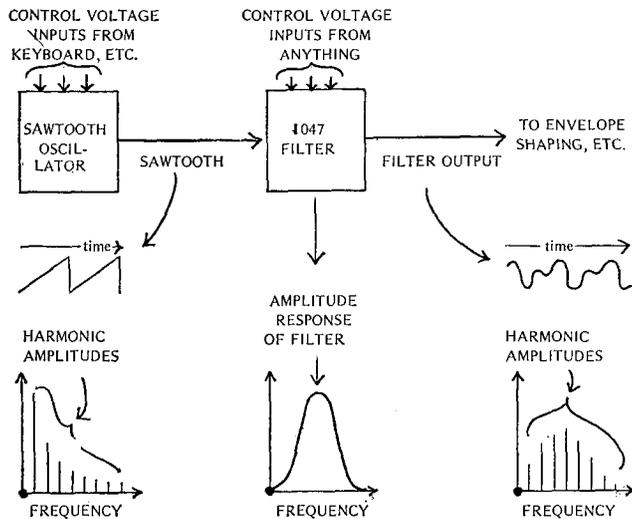


Fig. 1. Typical filter application in sound synthesis.

a reed, violin bow, or mechanical impulse, on a resonator which reinforces a region of the driving waveform's spectrum. Typical natural resonators are horns, strings, pipes, drums, and sounding boards. The 1047 filter's band-pass response is that of a natural acoustic resonator, namely, single pole (6 dB per octave), and has a resonance factor voltage controllable from 0 to 54 dB, that is, a  $Q$  range of  $1/2$  (damped) to 512.

**DESIGN**

The filter block diagram is shown in Fig. 2. The filter is basically an analog computing circuit consisting of summers and integrators, set up to solve a second-order differential equation. The circuit is well known and documented in analog computer and servomechanism fields.

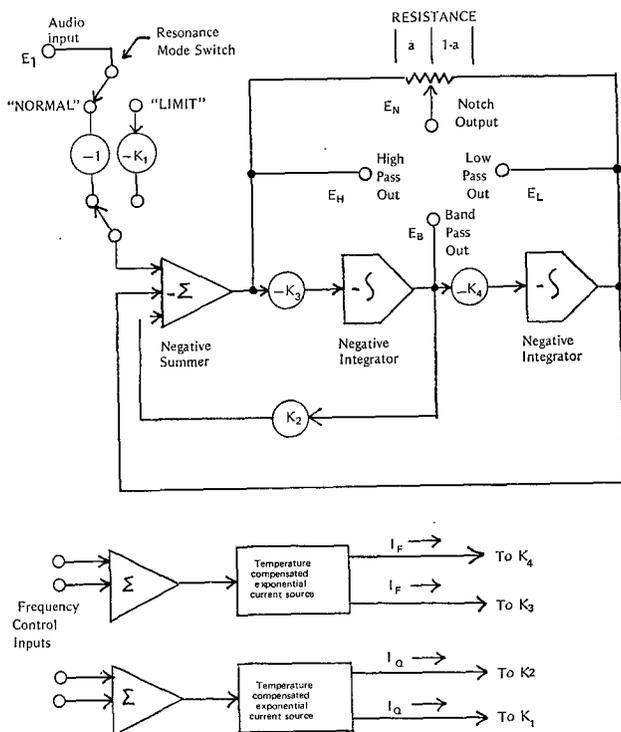


Fig. 2. Filter block diagram.

What is unique, however, is that the parameters ( $K_1$ ,  $K_2$ ,  $K_3$ , and  $K_4$ ) are voltage controlled by accurate temperature compensated exponential circuitry,<sup>2</sup> driving wide-range voltage-controlled amplifiers.

As will be shown in the mathematical analysis, the four  $K$  parameters are scaled so as to 1) provide independent control of  $F_c$  and  $Q$ , and 2) normalize the response so that the four outputs have unity gain in their respective passbands.

**MATHEMATICAL ANALYSIS**

The Laplace transform method will be used, so a few definitions are in order.

$s \equiv \sigma + j\omega$ , where  $\sigma$  is the damping factor (real axis) and  $\omega$  the angular frequency (imaginary axis)

$\omega \equiv 2\pi f$ , where  $f$  is the frequency in hertz

$1/s \equiv$  transfer function of an integrator

$j \equiv \sqrt{-1}$ .

Proceeding to the analysis, let us assume an input signal  $E_1$  (Fig. 2), which according to Laplace is an impulse (a voltage that jumps to infinite amplitude and back in zero time) of area equal to  $E_1$ . (I offer my sincere apologies to mathematicians who do not accept impulse functions.) Let us also assume the resonance mode switch to be in the "normal" position.

The output of the negative summer is  $E_H$ .  $E_B = E_H(K_3/s)$ ,  $E_L = E_H(K_3/s)(K_4/s)$ , so the summer's output  $E_H = E_1 - E_L - K_2E_B = E_1 - E_H(K_3K_4/s^2) - E_H(K_2K_3/s)$ . Multiplying by  $s^2$  and rearranging terms, and letting  $K_3 = K_4$ , we arrive at

$$E_H = E_1 \frac{s^2}{s^2 + K_2K_3s + K_3^2}, \quad \text{High-pass response}$$

$$E_B = E_H \left( \frac{K_3}{s} \right) = E_1 \frac{K_3s}{s^2 + K_2K_3s + K_3^2}, \quad \text{Band-pass response}$$

$$E_L = E_H \left( \frac{K_3^2}{s^2} \right) = E_1 \frac{K_3^2}{s^2 + K_2K_3s + K_3^2}, \quad \text{Low-pass response.}$$

If we now set the denominators equal to zero, the solutions for  $s$  are the poles (Fig. 3) of all three response functions:

$$s^2 + K_2K_3s + K_3^2 = 0,$$

$$s = \frac{-K_2K_3 \pm jK_3 \sqrt{1 - \frac{K_2^2}{4}}}{2} = \sigma_0 \pm j\omega_0.$$

The center frequency of the bandpass response  $\omega_c = \sqrt{\sigma_0^2 + \omega_0^2} = K_3$ . The resonance factor  $Q = -\omega_c / 2\sigma_0 = 1/K_2$ .  $K_3$  and  $K_2$  thus provide independent control of center frequency  $F_c$  and  $Q$ . Referring to Fig. 2, the notch output is a weighted sum of the high-pass and low-pass functions, that is, the notch response is

$$E_N = \frac{aK_3^2 + (1-a)s^2}{s^2 + K_2K_3s + K_3^2}.$$

<sup>2</sup> Covered by U. S. Patent 3444362.

# UNCONDITIONALLY STABLE COMBINATION VOLTAGE CONTROLLED FILTER/RESONATOR

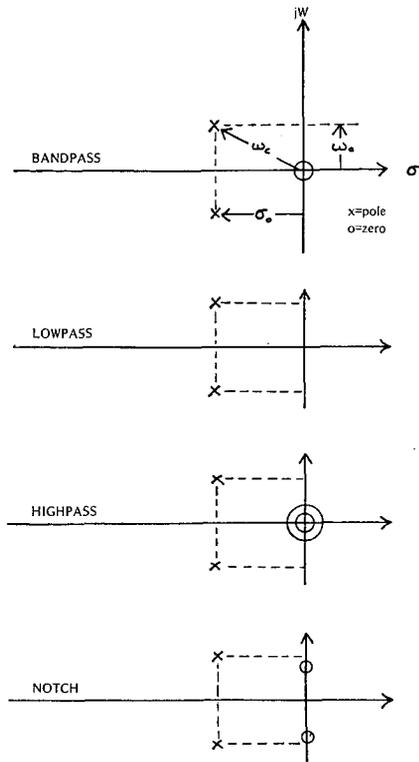


Fig. 3. Pole-zero locations for mathematical analysis.

Note that the notch response has the same poles as the other responses, but also has zeros at  $S = \pm jK_3 \sqrt{a/(1-a)}$ , which means that the notch frequency can be varied with respect to  $F_c$ .

If the resonance mode switch is in the "limit" position, the four responses will be scaled by  $K_1$ , which is made equal to  $K_2$ , resulting in a peak response of unity gain, regardless of  $Q$ .

## RESULTS

What has been achieved is a highly resonant filter with voltage-controlled frequency and resonance, that simul-

taneously provides high-pass, band-pass, low-pass, and notch outputs. The filter is capable of providing a wide variety of formant shaping and tonal modulation. The band-pass response is that of a natural acoustic resonator, and is most useful in synthesizing instrumental timbres. In addition, the high degree of stable resonance and frequency tracking accuracy attainable enables the filter to perform precise narrow-band spectrum analysis of audio signals (Figs. 4-6).

The center frequency  $F_c$  of the band-pass output is the cutoff frequency of the high-pass and low-pass outputs.  $F_c$  may be set by the coarse and fine frequency knobs over the range of 16 Hz to 16 kHz. Control signals applied to any  $F_c$  input will change the center frequency from the knob setting by 1 octave per volt when the knob above the control input is at maximum. Control signals from the individual inputs are summed with the  $F_c$  knob controls, and may be positive, negative, or audio.

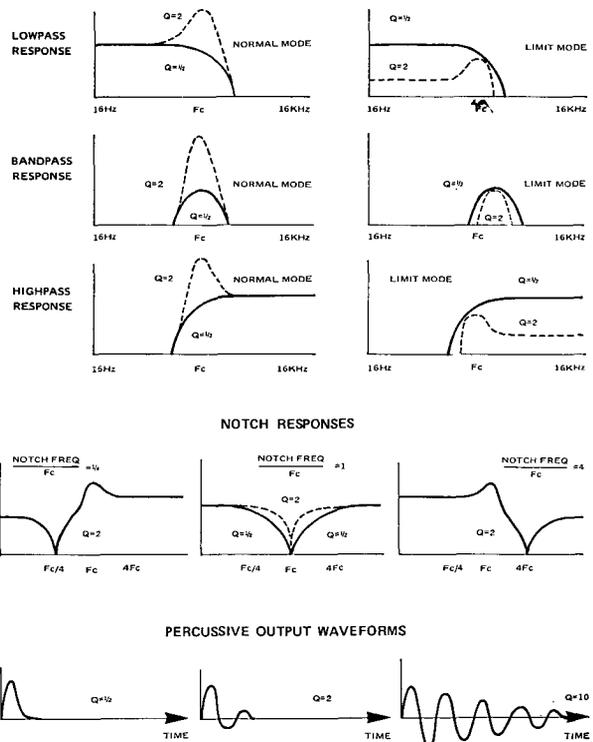


Fig. 5. Frequency and impulse responses.

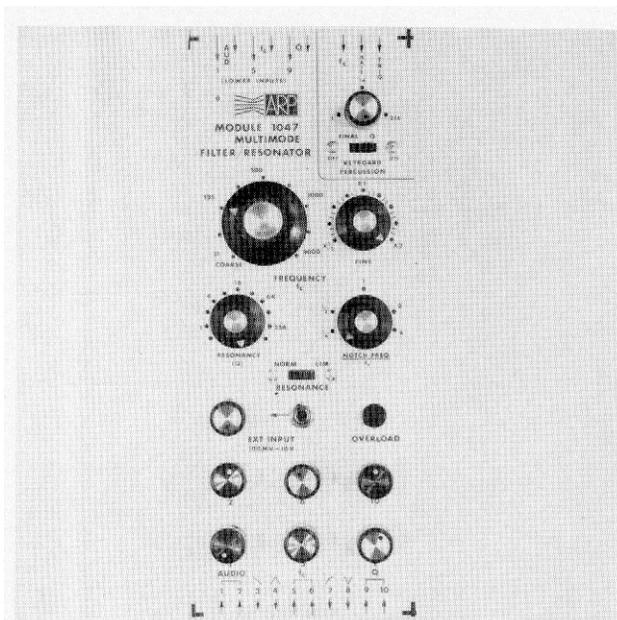


Fig. 4. Filter front panel (actual size).

With the resonance ( $Q$ ) knob at minimum and the resonance switch set to "norm," the band-pass output has a gain of 0.5 at  $F_c$  and attenuates 6 dB per octave above and below  $F_c$ . The low-pass output has unity gain from dc to  $F_c$  and attenuates 12 dB per octave above  $F_c$ . The notch output has flat response everywhere except for a deep (40-dB) notch at a frequency determined by the (notch frequency/ $F_c$ ) knob. With this knob set to 1, the notch occurs at  $F_c$ . Note that the notch output is effective only at low  $Q$ .

As the resonance ( $Q$ ) knob is turned up, a resonant peak occurs at  $F_c$  in all four outputs, except in the notch output when the notch frequency is at  $F_c$ . The gain at this peak is numerically equal to the  $Q$ , and the 3-dB bandwidth of this peak is equal to  $F_c/Q$ . Thus, as  $Q$  is varied from  $1/2$  to 512, the bandwidth varies from  $2 F_c$  (2 octaves) to  $F_c/512$  ( $1/32$  of a semitone). When using

CENTER FREQUENCY ( $F_c$ ):	16 Hz to 16 KHz, voltage controlled.
BANDPASS RESPONSE:	Single pole resonator, 6 db per octave.
RESONANCE ( $Q$ ):	$\frac{1}{2}$ to 512 (0 to 54 db peaking at $F_c$ ), voltage controlled.
BANDWIDTH (3 db):	2 octaves to 1/32 semitone.
HIGHPASS AND LOWPASS RESPONSE:	12 db per octave cutoff at $F_c$ , with same resonant peak at $F_c$ as in Bandpass Response.
NOTCH RESPONSE:	Resonant peak at $F_c$ as in Bandpass Response, plus notch at frequency determined by $\frac{NOTCH + FREQ}{F_c}$ control. With this control at 1, response is flat except for notch at $F_c$ .
NOTCH DEPTH:	> 40 db.
NOTCH WIDTH (3 db):	2 octaves to 1 semitone.
CONTROL INPUT RANGE:	$\pm 10$ v maximum.
INPUT IMPEDANCE:	50 K ohm minimum.
$F_c$ CONTROL CHARACTERISTIC:	1 octave per volt; at OV, $F_c$ is equal to the frequency knob setting.
$Q$ CONTROL CHARACTERISTIC:	1 volt doubles $Q$ ; at OV, $Q$ is equal to resonance ( $Q$ ) knob setting.
AUDIO INPUT RANGE:	$\pm 10$ v maximum.
AUDIO INPUT IMPEDANCE:	50 Kohm minimum.
AUDIO OUTPUT IMPEDANCE:	1 Kohm.
OVERLOAD LIGHT:	Indicates excessive input level.
KEYBOARD PERCUSSION:	Applies pulse from keyboard to filter, which rings according to $Q$ . Upon release of key, tone decays according to Final $Q$ knob setting.
POWER REQUIREMENTS:	$\pm 15$ volts @ 60 mA, regulated to $\pm 0.1\%$ . $+12$ to $+15$ volts @ 30 mA, lamp supply.

Fig. 6. Electrical specifications.

high resonance, the audio input controls may have to be turned down to prevent overload. An overload light is provided for this purpose. The  $Q$  may be controlled by external signals. The  $Q$  control characteristic is exponential, that is, each volt applied to a  $Q$  input doubles the  $Q$  when the input knob is at maximum.

With the resonance switch set to "lim," the height of the resonant peak is limited to unity gain at  $F_c$ , and the response on either side falls off as the  $Q$  is increased. This mode is useful when tuning sharply about a strong fundamental or harmonic of the input signal, but will otherwise result in a very low output signal at high resonance. For most applications, this switch should be set to "norm."

A low-level signal such as an electric organ or guitar may be plugged into the front panel ext input, which is mixed with the lower matrix switch audio inputs.

Upper matrix switch inputs for audio,  $F_c$  and  $Q$  are provided. The short arrows are independent, unattenuated inputs, while the long arrows, marked 1, 5, and 9, are wired directly to the corresponding lower inputs for the purpose of attenuating upper matrix switch inputs.

Another feature is keyboard percussion (Fig. 7), which allows the filter to generate a wide variety of percussive tones from the keyboard. The keyboard gate and trigger outputs should be connected to the gate and trigger inputs at the upper right corner of the panel, and the keyboard control voltage applied to any one  $F_c$  input. With the keyboard percussion switch on, striking a key produces a sharp percussive attack, followed by a tone which varies from a slightly pitched click resembling a castanet clap (at low  $Q$ ) to a slowly decaying sine tone at high  $Q$ . Upon releasing the key, the tone damps at a rate determined by the final  $Q$  knob. The band-pass output gives the most natural percussive quality, although the high-pass and low-pass outputs may be used. They give a sharper and a duller attack, respectively.

### MUSICAL APPLICATIONS: SYNTHESIS

Since the 1047 filter has a linear pitch versus control voltage function (1 octave per volt) and precise control of resonance, the device will track a voltage controlled oscillator. Some examples of synthesis applications are as follows.

1) Select any single harmonic up to at least the 30th from an oscillator, so that any note played will have this same harmonic emphasized.

2) Reject any harmonic in a likewise manner.

3) The filter may be controlled from a keyboard and fed from a fixed oscillator, so that harmonics may be selected and played as a scale (just intonation).

4) The filter may be fed white or pink noise and "played" on a keyboard, producing "pitched" noise. The degree of "pitchedness," or "color," may be remotely controlled over a range from no pitch sensation to a slowly but randomly varying sine wave (depending on  $Q$ ).

5) "Phasing," or "flanging," may be simulated by listening to the notch output and sweeping the center frequency  $F_c$ .

6) The simultaneous output capability permits the filter to be used as a voltage-controlled electronic crossover network.

7) Several filters may be combined in parallel or cascade to yield all sorts of complex timbral modulations and/or impulse responses.

8) By applying negative voltage to the frequency-control input, the center frequency may be driven down to at least 1 Hz, permitting it to modify transients such as square waves for envelope-shaping applications.

### MUSICAL APPLICATIONS: ANALYSIS

The 1047 can be used with an oscilloscope and sweep source (simple sawtooth oscillator) to perform real-time spectrum analysis of virtually any kind of input signal. The exponential frequency control characteristic yields a plot that is amplitude versus the logarithm of frequency, similar to conventional audio graph paper plots. By inserting a logarithmic converting circuit between the filter output and the oscilloscope vertical input, the plot

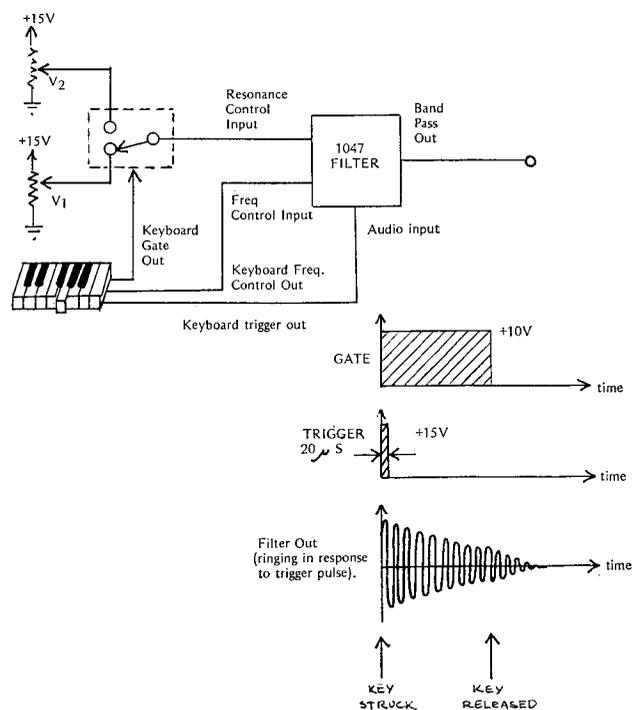


Fig. 7. Keyboard percussion as used in ARP synthesizer.

## UNCONDITIONALLY STABLE COMBINATION VOLTAGE CONTROLLED FILTER/RESONATOR

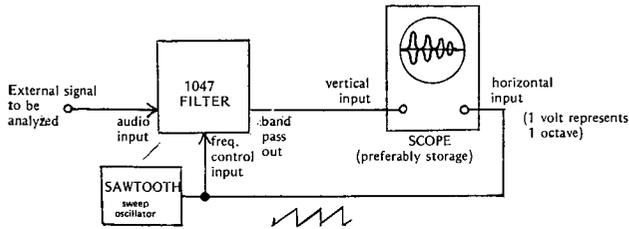


Fig. 8. Typical filter application in sound analysis (spectrum analyzer).

will be directly calibrated in decibels versus octaves (Fig. 8).

### MISCELLANEOUS APPLICATIONS

The range of applications for any voltage-controlled active filter is of course large, but a few that are particularly pertinent to the capabilities of the 1047 filter should be mentioned.

1) *Analog Computation*: The filter is basically a voltage-controlled analog computer, so it is useful in simulating and compensating feedback control systems.

2) *Biological Data Processing*: The filter can be used, for example, to isolate alpha waves from raw EEG outputs. For that matter, it can spectroanalyze the whole brain wave output in real time.

3) *Seismic and Vibration Measurement*.

4) *Speech Analysis and Synthesis*: Formants may be accurately controlled in a speech simulator, for example.

5) *Measurement*: The filter can be used to measure

frequency responses of circuits, loudspeakers, rooms, and anything that can be converted into an electrical signal.

6) *Distortion Analysis*: The filter can either measure the amplitude of each distortion component (harmonic or sideband, etc.), or used to notch out the fundamental sine wave and pass all the distortion at once.

7) *Art*: Fig. 9 shows an oscilloscope which simply crossplotted the high-pass and band-pass outputs of the filter, which was fed a mixture of square waves. The spirals are the polar equation of a damped sinusoid. The picture is merely an example of the infinite variety of "electronic art" than can be generated with the filter, an oscilloscope, and some experimentation.

8) *Education*: Music educators, physics instructors, psycho-acoustic researchers, in short, anyone who can use an accurate stable voltage-controlled resonator will find this filter an extremely useful tool to both explore and advance the state of their art.

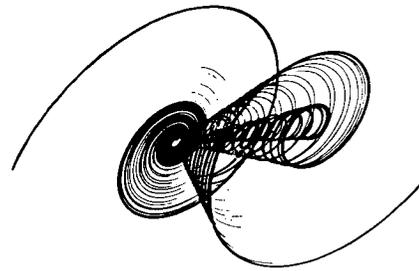
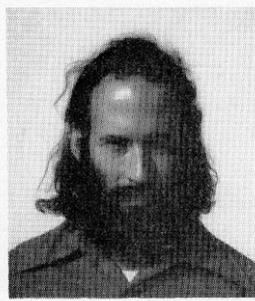


Fig. 9. Filter-oscilloscope art.

### THE AUTHOR



Dennis P. Colin was born in Urbana, Ill., on December 22, 1942. He received the B.S.E.E. degree from Lowell Technological Institute, Lowell, Mass., in 1967, ranking fourth in a class of 305. After graduation he joined the Raytheon Company's Missile Systems Division, Bedford, Mass., where he designed voltage-controlled 10 MHz phase shifters and other circuits for radar systems. Since 1969 Mr. Colin has been with

Tonus, Inc., Newton Highlands, Mass., as Director of Advanced Research and Development Engineering. He is responsible for most of the circuit design of the ARP synthesizers. Other inventions include voltage-controlled filters, oscillators, amplifiers, phase delay, modulators, envelope followers, frequency-to-voltage converters, frequency shifters, low-cost spectrum analysis systems, and other electronic music functions.